S475/1
SUBSIDIARY MATHEMATICS
AUG - 2014
2 hours 40 min

JINJA JOINT EXAMINATIONS BOARD
JJEB

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Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUG, 2014

SUBSIDIARY MATHEMATICS

Paper 1

2 hours 40 min

INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in **section A** and four questions from **section B.***

*All necessary working **must** be shown clearly*

Silent non-programmable scientific calculators an d mathematical tables with a list of formulae may be used.
In numerical work, take acceleration due to gravity g to be 9.8 ms^{-2}.

SECTION A (40 Marks)

Answer all questions in this section.

1. If $3log_t(10+3t) = 6$, find the value of t. ***(5 marks)***

2. Given the polynomial $P(x) = 2x^3 + ax^2 + bx - 36$ where $p(3) = 0$ and $P(2) = 26$. Find the values of a and b. ***(5 marks)***

3. The table below shows the number of text books owned by 11 students of a certain class and their total marks in an exam.

	A	B	C	D	E	F	G	H	I	J	K
Number of text books	5	8	2	9	7	5	3	10	1	4	6
Total marks	290	370	184	366	366	277	190	385	200	281	331

Calculate the coefficient of rank correlation between the number of text books and the total marks.

Comment on your result. ***(5 marks)***

4. Solve the equation $sec^2\theta - 2sec\theta + 15 = 0$ ***(5 marks)***

5. Three bags A, B and C, each contain fruits Mangoes, Oranges and Apples as follows;

	Bag A	Bag B	Bag C
Mangoes	3	2	5
Oranges	1	3	6
Apples	3	3	4

A bag is chosen at random and then a fruit is picked from the selected bag at random

Determine the probability that the fruit picked is

(i.) An orange

(ii.) Not an orange. ***(5 marks)***

6. A discrete random variable X is represented by the probability function

$$P(X=x) = \begin{cases} \frac{1+i}{ik} & ; i = 1,2,3,\ldots.6 \\ 0 & ; \ elsewhere \end{cases}$$

Find the ; (i.) value of k
(ii.) expectation of x ***(5 marks)***

7. A game coach has to choose a team of six out of ten of the best athletes, and one of the six has to be made a captain. How many possible teams can the coach obtain? *(5 marks)*

8. A hummer of mass 4.5 kg falls through a vertical height of 1 m and hits a nail of mass 50 grams directly without rebounding. If the nail is then driven into a piece of wood to a depth of 2 cm, find the common velocity of the hammer and nail just after impact. *(5 marks).*

SECTION B (60 MARKS)

Answer any four questions

9. (a.) The vectors $m = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $n = \begin{pmatrix} -6 \\ p \end{pmatrix}$ are perpendicular to each other. Determine;

(i.) The value of p

(ii.) $|2m + n|$ *(8 marks)*

(b.) Use matrix method to solve the simultaneous equations

$3x^2 + 5y = 2$

$2x^2 - 3y = 14$ *(7 marks)*

10. The gradient of curve at point P(x, y) is 4x + 3. If point A(3, 25) lies on the curve

(i.) Find the equation of the curve

(ii.) Determine the coordinates and nature of its turning point

(iii.) Find the area enclose by the curve and the x- axis. *(15 marks)*

11. The sales of soda (in crates) at a certain canteen, open five days a week, were as follows:-

week	Mon.	Tue.	Wed	Thur.	Fri.
1	142	177	213	171	138
2	125	172	191	170	131
3	114	158	192	155	127

(a.) Plot the data, together with the five- point moving averages.

(b.) Comment on the trend of the sales of soda

(c.) Estimate the sales (in crates) on Monday of the 4^{th} week. *(15 marks)*

12. (a.) if x is a normal random variable with mean 4 and variance 1.44. Find;

(i.) $p(x > 5.3)$

(ii.) $P(2.8 < x < 4.6)$ *(7 marks)*

(b.) The life time of a certain type of component of a computer is normally distributed with an average life of 1120 hours of working and standard deviation of 20 hours. Determine the probability that a component lasts

(i.) for less than 1150 hours

(ii.) Between 1115 hours to 1145 hours *(8 marks)*

13. The table below shows the age (at last birthday) at which expectant mothers in Jinja Hospital married

Age(in years)	16 – 20	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
Number of expectant mothers	60	120	80	30	3	20	10

(a.) Calculate

(i.) The mean age,

(ii.) The modal age , at which the expectant mothers married. *(7 marks)*

(b.) Draw an Ogive and use it to estimate;

(i.) The number of mothers who were 42 or over when they married.

(ii.) The number of mothers who were 18 or under when they were married.

(iii.) The range of the age of mothers for the middle 40% age bracket.

(8 marks)

14. A particle starts from rest, moving with a constant acceleration of 1.5 ms^{-1} for 30s.For the next 60s, the acceleration is 0.3 ms^{-1} and for the last 25s^{-1}, it decelerates uniformly to rest.

(a.) Sketch the velocity time graph for the motion of the particle *(5 marks)*

(b.) Find the acceleration of the particle during the last period of motion *(3 marks)*

(c.) Determine the total distance travelled by the particle *(4 marks)*

(d.) Find the average speed for the whole journey. *(3 marks)*

 End

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MARKING GUIDE S475/1 2014

1

$3\log_t(10+3t) = 6$.

$\log_t(10+3t)^3 = 6$. M1

$t^6 = (10+3t)^3$. M1

$t^2 = 10+3t$. B1

$t^2 - 3t - 10 = 0$. M1

$(t-5)(t+2) = 0$ A1 [05]

$t = 5$ or -2.

2.

$p(x) = 2x^3 + ax^2 + bx - 36$.

$p(3) = 9a + 3b - 18 = 0$. $3a + b = 6$

$p(2) = 4a + 2b - 20 = 26$. $3a + b = 6$ B1

$9a + 3b = 18$ ---(i) $2a + b = 23$ B1

$4a + 2b = 46$ ---(ii)

$3a + b = 6$

$2a + b = 23$ M1

$a = -17$. A1

$b = 57$. A1 [05]

3.

	A	B	C	D	E	F	G	H	I	J	K
Test Marks	5	8	2	9	7	5	3	10	1	4	6
Maths	290	370	184	366	368	277	190	385	200	251	331
R_{Test}	6.5	3	10	2	4	6.5	9	1	11	8	5
R_{Maths}	~~7~~ 6	~~8~~ 3	11	~~9.5~~ 4.5	~~9.5~~ 4.5	8	10	1	9	~~7~~ 2	~~5~~ 5
d	0.5	~~4~~ 1	1	~~1.5~~ 0.5	0.5	1.5	1	0	2	~~4~~ 1	~~0~~ 0
d^2	0.25	~~16~~ 1	1	~~6.25~~ 0.25	0.25	2.25	1	0	4	~~361~~ 1	~~0~~ 0

B1

$\Sigma d^2 = ~~53~~$ B1

$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \times ~~53~~}{11(11^2-1)} = ~~0.7636~~ 0.940$ M1 A1

comment → ~~positive~~ substantial correlation. B1 [05]

~~very high positive correlation.~~

4. $\sec^2\theta - 2\sec\theta - 15 = 0$.

$\sec\theta = \frac{2 \pm \sqrt{2^2 + 4\times1\times15}}{2}$ M1

$= 5 \text{ or } -3$ A1

$\sec\theta = 5$, $\sec\theta = -3$.

$\cos\theta = 0.2$, $\cos\theta = -0.3333$ M1

$\theta = 78.5°, 281.5°$, $\theta = 109.5°, 250.5°$.

$\therefore \theta = 78.5°, 281.5°, 109.5°, 250.5°$. A1 A1 [6]

5 $P(A) = \frac{1}{3}$ $P(B) = \frac{1}{3}$ $P(C) = \frac{1}{3}$.

$P(O/A) = \frac{1}{7}$ $P(O/B) = \frac{3}{8}$ $P(O/C) = \frac{6}{15}$. B1

(i) $P(O) = P(O \cap A) + P(O \cap B) + P(O \cap C)$

$= P(A)\cdot P(O/A) + P(B)\cdot P(O/B) + P(C)\cdot P(O/C)$

$= \frac{1}{3}\times\frac{1}{7} + \frac{1}{3}\times\frac{3}{8} + \frac{1}{3}\times\frac{6}{15}$. M1

$= \frac{1}{21} + \frac{1}{8} + \frac{2}{15}$.

$= \frac{257}{540} = 0.306$. A1

(ii) $P(\text{not orange}) = 1 - P(O)$

$= 1 - \frac{257}{540} = \frac{583}{540}$. M1

$= 0.694$ A1 [5]

6. $P(X=x) = \begin{cases} \frac{1+i}{ik}, & i=1,2,3\cdots 6 \\ 0, & \text{elsewhere} \end{cases}$

(i) $\sum_{i=0} P(X=x) = 1$.

$\frac{2}{k} + \frac{3}{2k} + \frac{4}{3k} + \frac{5}{4k} + \frac{6}{5k} + \frac{7}{6k} = 1$ M1

$\frac{1}{k}\left(\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6}\right) = 1$. M1

$\frac{1}{k} = \frac{20}{169}$.

$\therefore k = \frac{169}{20}$. B1

(ii) $E(x) = \sum x\, p(x=x)$.

$= \frac{20}{169}\left(2\times1 + \frac{3}{2}\times2 + \frac{4}{3}\times3 + \frac{5}{4}\times4 + \frac{6}{5}\times5 + \frac{7}{6}\times6\right)$ M1

$= \frac{20}{169}(2+3+4+5+6+7)$.

$= \frac{544}{169} = 3.195$. B1

[05]

7. Choosing 6 from 10.

$= {}^{10}C_6$. M1

choosing 1 captain from 6

$= {}^{6}C_1$. M1

Number of team $= {}^{10}C_6 \times {}^{6}C_1$ M1

$= 210 \times 6 = 1260$ M1 B1

[05]

8. $M_1 = 4.5kg$, $M_2 = 50g = 0.05kg$.

$H_1 = 1m$, $H_2 = 2cm = 0.02m$.

$M_1V_1 + M_2V_2 = (M_1+M_2)V$. B1

$M_1gh = \frac{1}{2}M_1V_1^2$.

$9.8 \times 1 = \frac{1}{2}V_1^2$. M1

$V_1 = \sqrt{19.6} = 4.427 ms^{-1} \rightarrow V_2 = 0$ A1

$4.5 \times 4.427 + 0.05 \times 0 = (4.5+0.05)V$. M1

$V = \frac{19.9215}{4.55} = 4.378 ms^{-1}$ A1

05

SECTION B.

9. (a) $\underset{\sim}{M} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\underset{\sim}{n} = \begin{pmatrix} -6 \\ p \end{pmatrix}$.

(i) $\underset{\sim}{M} \cdot \underset{\sim}{n} = 0$.

$\begin{pmatrix} 5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ p \end{pmatrix} = 0$. M1

$-30 + 3p = 0$. M1

$3p = 30$

$p = 10$. A1 A1

(ii) $|2\underset{\sim}{M} + \underset{\sim}{n}| = 2\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -6 \\ 10 \end{pmatrix} = \begin{pmatrix} 4 \\ 16 \end{pmatrix}$. M1 A1

$|2\underset{\sim}{M} + \underset{\sim}{n}| = \sqrt{4^2 + 16^2}$. M1

$= \sqrt{272} = 16.49$ A1

10. (a) $\frac{dy}{dx} = 4x + 3$.

(i) $y = \int 4x + 3 \, dx$. M1

$y = 2x^2 + 3x + C$. A1

$A(3, 25)$. M1

$25 = 2 \times 3^2 + 3 \times 3 + C$. A1

$C = -2$. B1

$\therefore y = 2x^2 + 3x - 2$.

(ii) $\frac{dy}{dx} = 4x + 3 = 0$ M1

$4x = -3$

$x = -\frac{3}{4}$. A1

	$-\frac{3}{4}$	
−		+

min. M1

$y = 2 \times \left(\frac{-3}{4}\right)^2 + 3 \times \frac{-3}{4} - 2$. M1

$= \frac{18}{16} - \frac{9}{4} - 2 = -3\frac{1}{8}$. A1

$\left(-\frac{3}{4}, -\frac{25}{8}\right)$ Minimum B1

(iii) $2x^2 + 3x - 2 = 0$.

$(x + 2)(2x - 1) = 0$

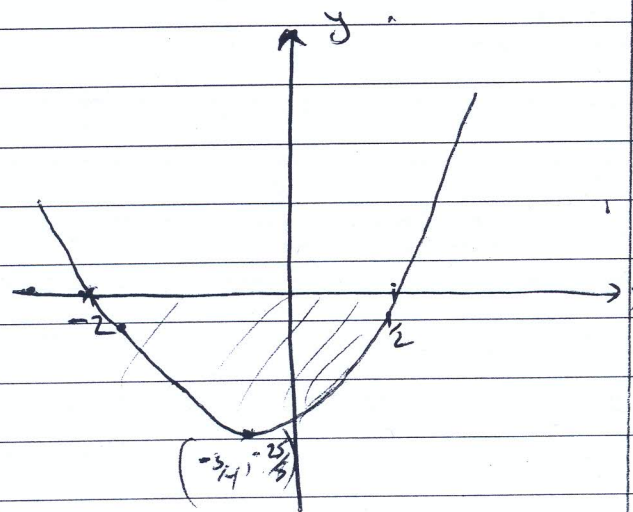
$x = -2, \frac{1}{2}$. B1 (Sketch)

Area $= \int_{-2}^{\frac{1}{2}} (2x^2 + 3x - 2) \, dx$.

$= \left[\frac{2}{3}x^3 + \frac{3}{2}x^2 - 2x\right]_{-2}^{\frac{1}{2}}$. M1

$= \frac{13}{24} - \frac{14}{3}$ M1

$= \frac{33}{8} = 4\frac{1}{8}$ A1 [15]



(b) $3x^2 + 5y = 2$

$2x^2 - 3y = 14$.

$$\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x^2 \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}.$$ B1

$$\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}^{-1} = \frac{1}{-19}\begin{pmatrix} -3 & -5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3/19 & 5/19 \\ 2/19 & -3/19 \end{pmatrix}.$$ M1 A1 (for inverse)

$$\begin{pmatrix} 3/19 & 5/19 \\ 2/19 & -3/19 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} x^2 \\ y \end{pmatrix} = \begin{pmatrix} 3/19 & 5/19 \\ 2/19 & -3/19 \end{pmatrix}\begin{pmatrix} 2 \\ 14 \end{pmatrix}.$$ M1

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x^2 \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$ M1

$$\begin{pmatrix} x^2 \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}.$$

$x^2 = 4$ $\qquad y = -2$. A1

$x = \pm 2$ A1

15

11.

WEEK	DAY	SALES	FIVE POINT MOVING TOTALS	FIVE POINT MOVING AVERAGES
1	MON	142		
	TUE	177		
	WED	213	841	168.2
	THUR	171	824	164.8
	FRID	138	819	163.8
2	MON	125	797	159.4
	TUE	172	796	159.2
	WED	191	789	157.8
	THUR	170	778	155.6
	FRID	131	764	152.8
3	MON	114	765	153
	TUE	158	750	150
	WED	192	746	149.2
	THUR	155		
	FRID	127		

B_6 (B_1 for every 2 correct)

(b) The sales declined gradually over the three weeks. B_1

(c) Let monday sales be x.

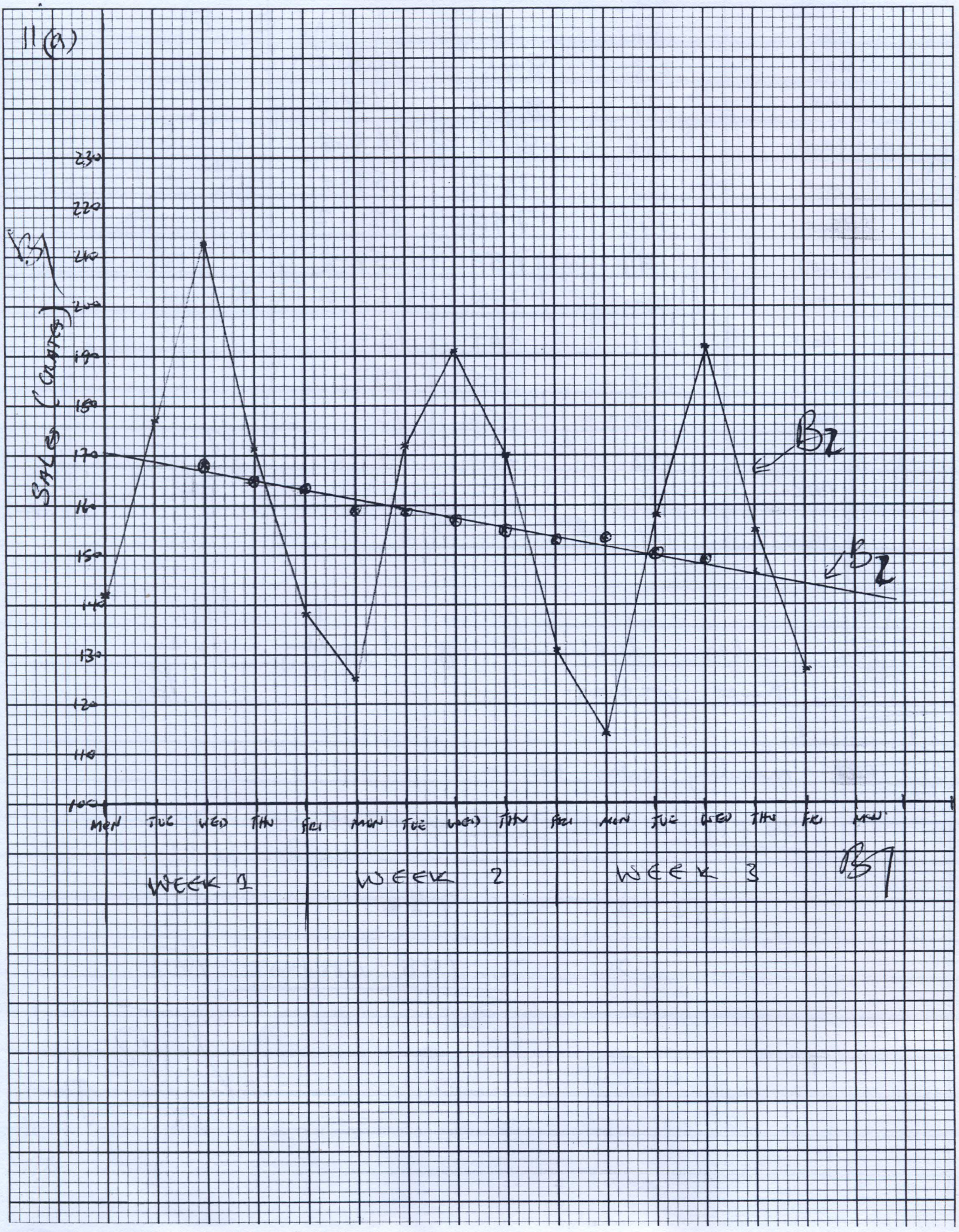
$$\frac{158 + 192 + 155 + 127 + x}{5} = 148 \quad M_1$$

$$632 + x = 740$$

$$x = 108 \text{ crates} \quad B_1$$

12

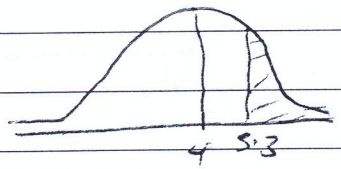
11(a)


SALES (CLAPS)
230
220
210
200
190
180
170
160
150
140
130
120
110
100
MON TUE WED THU FRI MON TUE WED THU FRI MON TUE WED THU FRI MON
WEEK 1
WEEK 2
WEEK 3
B1
B2
B3

12(a) $\bar{x} = 4$, $\sigma^2 = 1.44$.

$\sigma = 1.2$. B1

(i) $P(x > 5.3)$

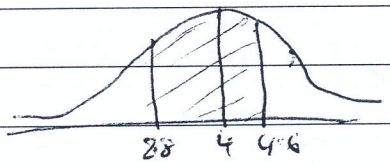


Standardizing. $z = \frac{x - \bar{x}}{\sigma} = \frac{5.3 - 4}{1.2}$. M1

$= 1.0833$.

$P(z > 1.0833) = 0.5 - 0.360 = 0.14$. M1 A1

(ii) $P(2.8 < x < 4.6)$.



Standardizing. $z_1 = \frac{2.8 - 4}{1.2} = -1$ M1

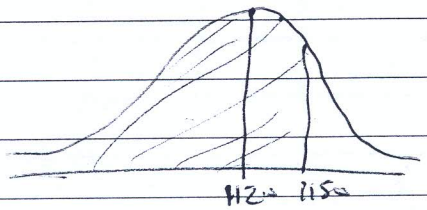
$z_2 = \frac{4.6 - 4}{1.2} = 0.5$.

$P(-1 < z < 0.5) = 0.341 + 0.191$ M1

$= 0.532$ A1

(b) $\bar{x} = 1120$ hours, $\sigma = 20$ hours. B1

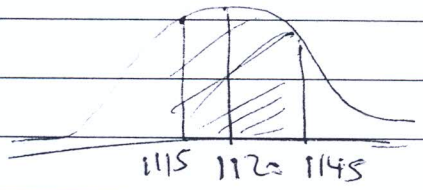
(i) $P(x < 1150)$.



Standardizing. $z = \frac{1150 - 1120}{20} = 1.5$ M1

$P(Z < 1.5) = 0.933.$ M1A1

(iii) $P(1115 < x < 1145)$



standardising $z_1 = \frac{1115 - 1120}{20} = -0.25$ M1

$z_2 = \frac{1145 - 1120}{20} = 1.25.$ M1

$P(-0.25 < z < 1.25) = 0.199 + 0.394$ M1

$= 0.593.$ A1

[5]

B.

AGE	Freq.	x	fx	Class boundaries	Cum. freq
16–20	60	18	1080	16 – 20	60
20–24	120	22	2640	20 – 25	180
25–29	50	27	2160	25 – 30	260
30–34	30	32	960	30 – 35	290
35–39	3	37	111	35 – 40	293
40–44	20	42	840	40 – 45	313
45–49	10	47	470	45 – 50	323 B1
	323		8261 B1	B1	

(i) Mean $= \frac{\Sigma fx}{\Sigma f}$

$= \frac{8261}{323} = 25.58.$ M1 A1

(ii) modal age = $L_0 + \left(\frac{d_1}{d_1+d_2}\right) c$

$= 20 + \left(\frac{60}{60+40}\right) 5$ M1

$= 23$ B1

(b) (i) 42 and over.

$= 300$ women. B1

(ii) 18 and under

$= 37$ women. B1

(iii) Middle 40%

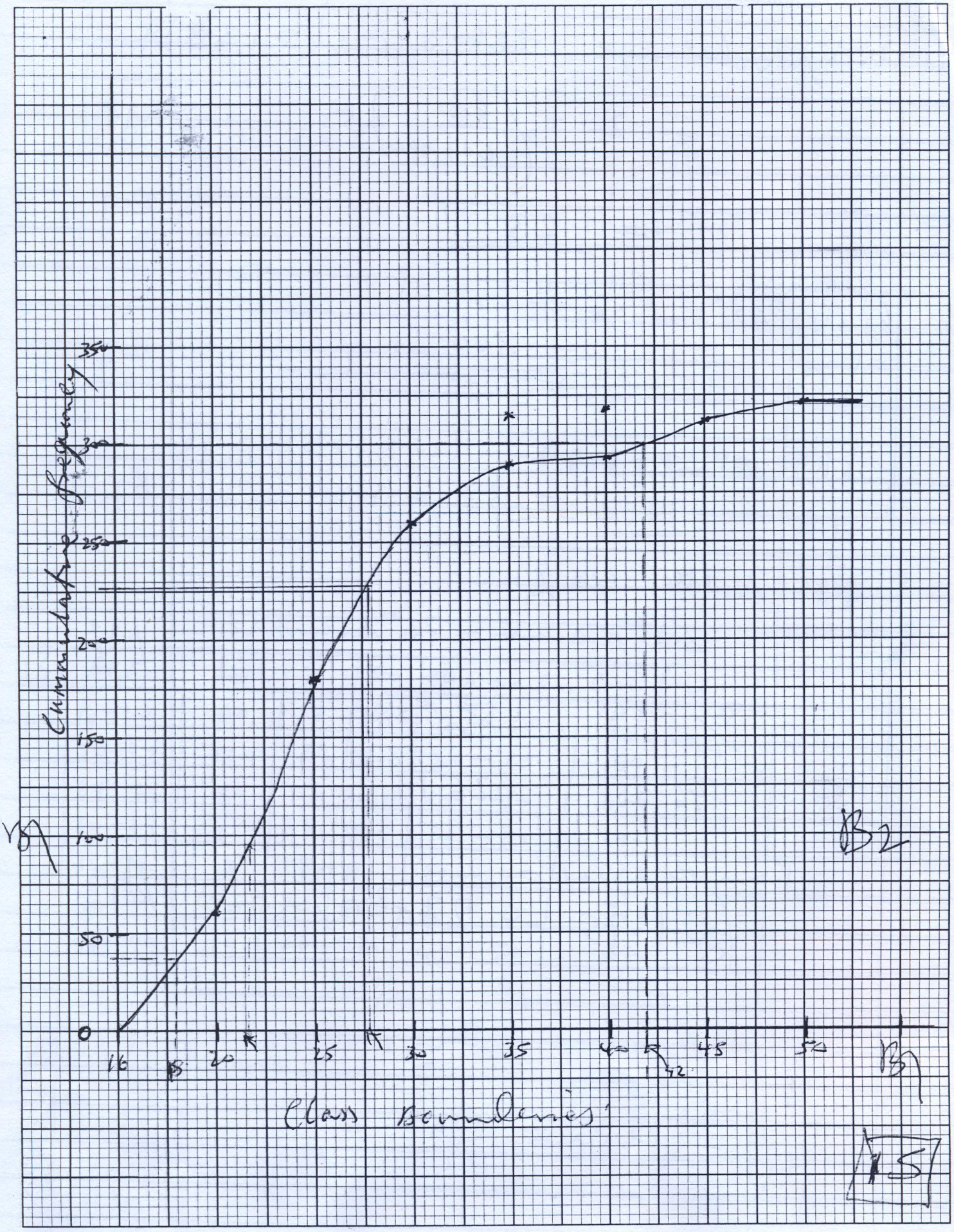
30th percentile – 70th percentile. M1

$\frac{30}{100} \times 323$, $\frac{70}{100} \times 323$.

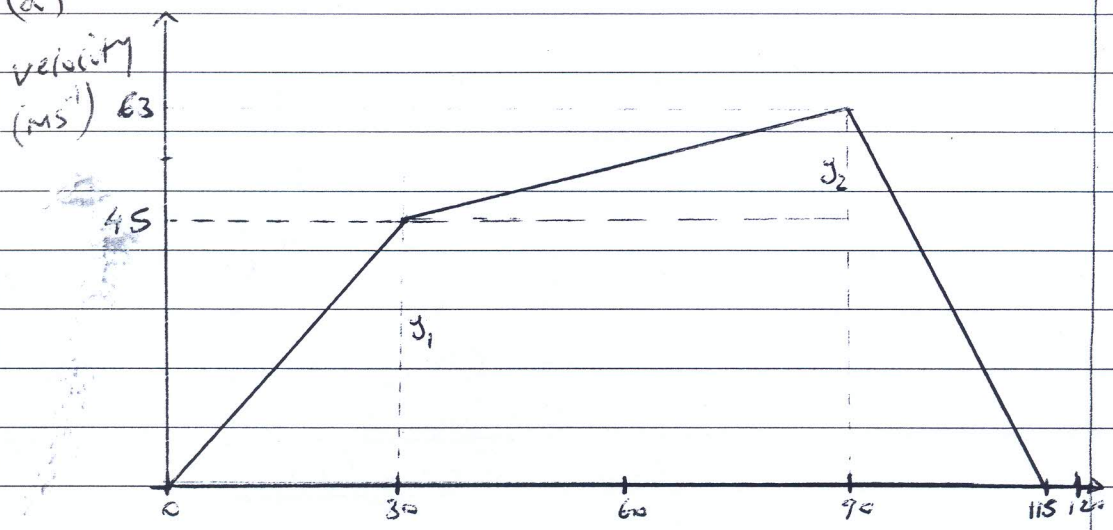
96.9 , 226.1

from the graph.

(21.75 — 27.75) years B1


Cumulative frequency
350
300
250
200
150
100
50
0
16
20
25
30
35
40
42
45
50
Class boundaries
B2

14 (a)



B1 B1 B1 B1 (axes)

$y_1 = 1.5 \times 30 = 45$. M1

$y_2 = 0.3 \times 60 = 18$. M1

$y_1 + y_2 = 45 + 18 = 63$. B1

(b) Acceleration during last period

$= \frac{-63}{25}$. M1

$= -2.52 \text{ ms}^{-2}$. B1

(c) Total distance travelled

= Area under curve.

$= \frac{1}{2} \times 30 \times 45 + \frac{1}{2} \times 60 (45+63) + \frac{1}{2} \times 25 \times 63$ M1 M1 M1

$= 675 + 3240 + 787.5$.

$= 4702.5 \text{ M}$ B1

(d) Average Speed $= \frac{\text{Total distance}}{\text{Total Time}}$

$= \frac{4702.5}{115}$ M1

$= 40.89 \text{ ms}^{-1}$ A1 [15]

- END -